Regularization of the Density of States Fluctuation Contribution in Magnetic Field

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Abstract

The fit of the experimental data on c-axis magnetoresistance of HTS above the transition temperature with the theory based on the fluctuation renormalization of the one-electron density of states (DOS) is exellent in weak magnetic fields but meets the noticible difficulties in the region of strong fields. This is due to the formal divergency of the DOS contribution to conductivity and the dependence of the cut-off parameter on the magnetic field itself. We propose the scheme of the regularization of the problem. This permits us to obtain the expression for the magnetic field dependent part of DOS conductivity as a convergent serie independent on cut-off. We also calculate analitically the asymptotics for all regions of magnetic fields. The results demonstrate the robustness of the DOS contribution with respect to the magnetic field effect: in strong fields ($H > H_{c2}(T-T_c)$) it decreases logarithmically only while Aslamazov-Larkin and anomalous Maki-Thompson contributions diminish as powers of $\frac{H_{c2}}{H}$.

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The recently proposed idea of the importance of the one-electron density of states (DOS) renormalization due to superconducting fluctuations was successfully applied to the explanation of the c-axis resistance temperature dependence, optical conductivity, c-axis magnetoresistance, NMR rate behavior near T_c, tunneling conductance and other normal state anomalies of HTS in metallic part of the HTS phase diagram [1]. It turns out that the strong renormalization of the DOS in a very narrow vicinity of the Fermi level ($E \sim T - T_c$) together with the particle conservation law leads to the observation of the pseudogap-like phenomena in a set of normal state characteristics of HTS. The prediction [2, 3] of the sign change of the c-axis magnetoresistance was especially interesting and later this effect was observed on several experiments [4, 5]. The fit of these experimental data with the theory based on the fluctuation renormalization of the one-electron density of states [6, 3] is excellent for weak magnetic fields but meets the noticeable difficulties in the region of strong fields. This is due to the formal divergency of the DOS contribution to conductivity and the dependence of the cut-off parameter on the magnetic field itself [3]. The similar problem arises for the DOS contributions to NMR and other physical properties measured in magnetic field.

In this communication we clarify the problem of the regularization of DOS contribution in an arbitrary magnetic field on the example of c-axis magnetoconductivity of quasi-two-dimensional superconductor looking forward to apply the proposed theory to the measurements on HTS compounds. This permits us to leave apart in this communication the discussion of the Aslamazov-Larkin and anomalous Maki-Thompson contributions, which were studied in details (see, for example, [7]) but due to the low interlayer transparency of the most of HTS materials they may be omitted beyond the immediate vicinity of T_c. Below we imply the model of electron spectrum in the form of corrugated cylinder and use notations introduced in [3], where the tensor of fluctuation conductivity of layered superconductor in magnetic field was discussed. The magnetic field is supposed to be oriented along c-axis, so finally the problem is reduced to the carrying out of the summation over the Landau states of the center of mass of fluctuation Cooper pair [3].To avoid the problem of the ultraviolet divergence of the DOS contribution with the badly defined cut-off depending on the magnetic field [3] we calculate the cut-off independent difference of magnetoconductivity [8]in field and its absence:

$$\Delta\sigma_c^{DOS}(h,\epsilon) = \sigma_c^{DOS}(h,\epsilon) - \sigma_c^{DOS}(0,\epsilon), \tag{1}$$

where $\epsilon = \ln T/T_c = \frac{T-T_c}{T_c}$ is reduced temperature, $h = \frac{H}{H_{c2(0)}}$ is the dimensionless magnetic field (both these parameters are supposed to be small: $\epsilon, h \ll 1$).

For this purpose the zero-field value $\sigma_c^{DOS}(0,\epsilon)$ [3] may be rewritten in the form:

$$
\begin{aligned}
\sigma_c^{DOS}(0,\epsilon) &= -\lim_{h\to 0}\frac{e^2 sr\kappa}{8\eta}h\int_{-1/2}^{1/h+1/2}\frac{dn}{\sqrt{\epsilon+(2n+1)h}\sqrt{\epsilon+r+(2n+1)h}} = \\
&= -\lim_{h\to 0}\frac{e^2 sr\kappa}{8\eta}h\sum_{\mathbf{n=0}}^{\mathbf{1/h}}\int_{-1/2}^{1/2}\frac{dx}{\sqrt{\epsilon+(2n+1/2+x)h}\sqrt{\epsilon+r+(2n+1/2+x)h}} \\
&= -\lim_{h\to 0}\frac{e^2 sr\kappa}{8\eta}\sum_{\mathbf{n=0}}^{\mathbf{1/h}}\ln\frac{\sqrt{\epsilon+2nh+2h}+\sqrt{\epsilon+r+2nh+2h}}{\sqrt{\epsilon+2nh}+\sqrt{\epsilon+r+2nh}}, \qquad (2)
\end{aligned}
$$

where $r = \frac{4\xi_c^2(0)}{s^2}$ is the dimensionless anisotropy parameter of the Laurence-Doniach model (which is supposed to be small: $r \ll 1$), s is the interlayer distance, η is the gradient coefficient of the $2D$ Ginzburg-Landau theory. Here

$$
\begin{aligned}
\kappa &= \frac{-\psi'(\frac{1}{2}+\frac{1}{4\pi\tau T})+\frac{1}{2\pi\tau T}\psi''(\frac{1}{2})}{\pi^2[\psi(\frac{1}{2}+\frac{1}{4\pi\tau T})-\psi(\frac{1}{2})-\frac{1}{4\pi\tau T}\psi'(\frac{1}{2})]} \\
&\to \begin{cases} 56\zeta(3)/\pi^4 \approx 0.691, & \text{for } T\tau << 1, \\ 8\pi^2(\tau T)^2/[7\zeta(3)] \approx 9.384(\tau T)^2, & \text{for } T\tau >> 1 \end{cases} \qquad (3)
\end{aligned}
$$

is a function of τT only [3]. It appears as the result of summation of all DOS type diagrams.

Let us substitute the expression (2) in (1). Now for the difference $\sigma_c^{DOS}(h,\epsilon) - \sigma_c^{DOS}(0,\epsilon)$ we may write the following formula, where the summation may be extended until $N \longrightarrow \infty$ due to good convergence of the sum (the n-th term of the sum is proportional to $n^{-3/2}$ for large n):

$$
\begin{aligned}
\Delta\sigma_c^{DOS}(h,\epsilon) &= \\
&= \frac{e^2 sr\kappa}{8\eta}h\sum_{\mathbf{n=0}}^{\infty}\{\frac{1}{h}\ln\frac{\sqrt{\epsilon+2nh+2h}+\sqrt{\epsilon+r+2nh+2h}}{\sqrt{\epsilon+2nh}+\sqrt{\epsilon+r+2nh}} - \\
&\quad -\frac{1}{\sqrt{\epsilon+2nh+h}\sqrt{\epsilon+r+2nh+h}}\} \qquad (4)
\end{aligned}
$$

This expression is very suitable for numerical calculation to analyze experimental data. It permits to obtain easily the asymptotic behavior of magnetoconductivity in the case of non-weak fields (note the inaccuracy of the analysis of this asymptotics in [3]). The case of very strong fields $h \gg \max\{\epsilon, r\}$, in contrast to [3], becomes now trivial for consideration: with logarithmic accuracy it is determined just by the contribution of the first term in (4) :

$$\Delta\sigma_c^{DOS}(h \gg \max\{\epsilon, r\}) = \frac{e^2 sr\kappa}{8\eta} \ln \frac{\sqrt{2h}}{\sqrt{\epsilon} + \sqrt{\epsilon + r}}. \tag{5}$$

The further analysis of (4) shows that for intermediate fields in the temperature range of three dimensional fluctuations ($\epsilon \ll h \ll r$) some nontrivial, typically 3D, behavior $\sim \sqrt{h}$ can be obtained:

$$\Delta\sigma_c^{DOS}(\epsilon \ll h \ll r) = \frac{e^2 sr\kappa}{16\eta} \sum_{n=0}^{\infty} \{\ln \frac{1 + \sqrt{\frac{h}{r}}\sqrt{2n+2}}{1 + \sqrt{\frac{h}{r}}\sqrt{2n}} - \tag{6}$$

$$-\sqrt{\frac{h}{r}} \frac{1}{\sqrt{2n+1}}\} = 0.42 \frac{e^2 sr\kappa}{8\eta} \sqrt{\frac{h}{r}}.$$

The limit of weak fields $h \ll \epsilon$ turns out to be a too cumbersome in the presentation (4). Then the simplest way to obtain the asymptotic is to use the Euler-McLaurin formula to transform the sum into an:

$$\Delta\sigma_c^{DOS}(h \ll r, \epsilon) = \frac{e^2 sr\kappa}{192\eta_2} \frac{2\epsilon + r}{[\epsilon(\epsilon + r)]^{3/2}} h^2. \tag{7}$$

In addition to the DOS contribution it is necessary to take into account the regular Maki-Thompson contribution, which in the case of weak fields takes form [3]:

$$\Delta\sigma_c^{MT(reg)}(h \ll r, \epsilon) = \frac{e^2 sr\tilde{\kappa}}{192\eta} \frac{r}{[\epsilon(\epsilon + r)]^{3/2}} h^2, \tag{8}$$

where

$$\begin{aligned} \tilde{\kappa} &= \frac{-\psi'(\frac{1}{2} + \frac{1}{4\pi T\tau}) + \psi'(\frac{1}{2}) + \frac{1}{4\pi T\tau}\psi''(\frac{1}{2})}{\pi^2[\psi(\frac{1}{2} + \frac{1}{4\pi\tau T}) - \psi(\frac{1}{2}) - \frac{1}{4\pi\tau T}\psi'(\frac{1}{2})]} \\ &\to \begin{cases} 28\zeta(3)/\pi^4 \approx 0.3455 & \text{for } T\tau << 1, \\ \pi^2/[14\zeta(3)] \approx 0.5865 & \text{for } T\tau >> 1 \end{cases} \end{aligned} \tag{9}$$

is another function of the impurities concentration only. We note that the regular MT term has the same sign as the overall DOS contribution. In weak fields it becomes noticeable only in 3D case $\epsilon \ll r$ and in the dirty limit when $\tilde{\kappa}$ is comparable with κ.

The evaluation of the regular Maki-Thompson contribution to magneto-conductivity may be done by the same procedure as in (2) for the analysis of non-weak fields :

$$\begin{aligned}\Delta\sigma_c^{MT(reg)}(h,\epsilon) &= -\frac{e^2 s\tilde{\kappa}}{4\eta_2} h \sum_{n=0}^{\infty}\{\frac{\epsilon+(2n+1)h+r/2}{\sqrt{\epsilon+(2n+1)h}\sqrt{\epsilon+r+(2n+1)h}} - \\ & -\frac{1}{h}[\sqrt{\epsilon+(2n+3/2)h}\sqrt{\epsilon+r+(2n+3/2)h} - \\ & \sqrt{\epsilon+(2n+1/2)h}\sqrt{\epsilon+r+(2n+1/2)h}]\}\end{aligned} \tag{10}$$

For the 3D case in the region of intermediate fields it leads to

$$\begin{aligned}\Delta\sigma_c^{MT(reg)}(\epsilon \ll h \ll r) &= \frac{e^2 s\tilde{\kappa}}{4\eta}\sqrt{\frac{h}{r}}\sum_{n=0}\{\sqrt{2n+3/2} - \\ & -\sqrt{2n+1/2} - \frac{1}{2\sqrt{2n+1}}\} = 0.02\frac{e^2 s\tilde{\kappa}}{4\eta}\sqrt{\frac{h}{r}}.\end{aligned} \tag{11}$$

One can see that the contribution $\Delta\sigma_c^{MT(reg)}(\epsilon \ll h \ll r)$ can be of the same order as $\Delta\sigma_c^{DOS}(\epsilon \ll h \ll r)$ (see (6)) depending on the relation between parameters $0.1\tilde{\kappa}$ and $r\kappa$; in dirty case it certainly has to be taken into consideration.

The analysis of (10) in the strong filed case ($h \gg \max\{\epsilon, r\}$) shows that

$$\begin{aligned}\Delta \quad \sigma_c^{MT(reg)} \quad & (h \gg \max\{\epsilon, r\}) = \\ = \quad & \frac{\pi^2 e^2 s\tilde{\kappa}}{128\eta}\frac{1}{h}\{\epsilon(\epsilon+r) + \frac{2}{\pi^2}(|\psi(1/4)| - |\psi(3/4)| - \frac{\pi^2}{4})r^2\} \\ = \quad & \frac{\pi^2 e^2 s\tilde{\kappa}}{128\eta}\cdot\frac{\epsilon(\epsilon+r)+0.136r^2}{h}\end{aligned} \tag{12}$$

and it evidently can be omitted in comparison with (5).

Let us analyze the results obtained starting from the 2D case $r \ll \epsilon$. The positive DOS contribution in magnetoconductivity growth as H^2 up to $H_{c2}(\epsilon)$ (see (7)) and then the crossover to a slow logarithmic asymptotic takes place. One can notice that at $H \sim H_{c2}(0)$ the value of $\Delta\sigma_c^{DOS}(h \sim 1, \epsilon) = -\sigma_c^{DOS}(0, \epsilon)$ what means the total suppression of the fluctuation correction in such a strong field. The regular part of the Maki-Thompson contribution does not manifest itself in this case.

In 3D case ($\epsilon \ll r$) the behaviour of magnetoconductivity is mainly the same besides the existence of the intermediate region $\epsilon \ll h \ll r$ with $\sqrt{h}$ dependence on magnetic field. Two crossovers take place here: from h^2 to $\sqrt{h}$ (at $h \sim \epsilon$) and from $\sqrt{h}$ to $\ln h$ (at $h \sim r$) behaviour. It is important that in this case the regular part of the Maki- Thompson contribution becomes to be comparable with DOS contribution or even dominating on the latter (in the case of dirty and very anisotropic superconductor). This domination anyway terminates at strong fields $h \succeq r$ where $\Delta\sigma_c^{MT(reg)}(h, \epsilon)$ rapidly decreases $\sim \frac{r}{h}$ with the field increase in contrast to the robust $\Delta\sigma_c^{DOS}(h, \epsilon) \sim \ln \frac{h}{r}$ which survives up to $h \sim 1$.

It worth to mention that the consideration presented above is valid not only for c-axis magnetoconductivity but with minimal variations in the DOS contribution coefficient, for any physical value like in-plane magnetoconductivity, tunneling conductivity, NMR rate, Hall effect etc. In these problems the DOS contribution certainly has to be treated side by side with the AL and anomalous MT ones (if there are no special reasons for their suppression, like in the case of c-axis transport for both of them, in the case of NMR for AL contribution only, or in the case of d-pairing for the MT one [1]).

It is important to stress that the suppression of DOS by magnetic field contribution takes place very slowly. Such robustness with respect to the magnetic field is of the same physical origin as the slow logarithmic dependence of the DOS-type corrections on temperature. This differs strongly the DOS contribution from Aslamazov-Larkin and Maki-Thompson ones [9], making the former noticeable in the wide range of temperatures (up to $\sim 2 \div 3T_c$) and magnetic fields ($\sim H_{c2}(0)$). The scale of the suppression of DOS contribution can be treated as the value of the pseudogap observed in the experiments mentioned above [1]. It has the order of $\Delta_{pseudo} \sim 2 \div 3T_c$ for magnetoconductivity and NMR, $\Delta_{pseudo} \sim \pi T_c$ for tunneling and $\Delta_{pseudo} \sim \tau^{-1}$ in optical conductivity.

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